Booster, AGS, and RHIC Parameters for the 2003–2004 RHIC Run

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The Tables in this note contain the nominal parameter values for the 2003–2004 RHIC Run.

1 Basic Formulae

1.1 Mass, Energy, Momentum, Rigidity, and Frequency

A Gold ion with charge eQ has $N = 197$ Nucleons, $Z = 79$ Protons, and $(Z - Q)$ electrons. (Here Q is an integer and e is the charge of a single proton.) The mass and energy are

$$m = au - Qm_e + E_b/c^2, \quad E = \sqrt{p^2c^2 + m^2c^4} \tag{1}$$

where $a = 196.966552$ is the atomic mass [1, 2] of the neutral Gold atom, $u = 931.494013$ MeV/c^2 is the unified atomic mass unit [3], $m_ec^2 = .510998902$ MeV is the electron mass [3], and p is the momentum. E_b is the binding energy of the Q electrons removed from the neutral Gold atom. This amounts to 0.327 MeV for the fully stripped Gold ion as calculated by Trbojevic [4]. The kinetic energy is defined to be

$$W = E - mc^2. \tag{2}$$

In terms of W, the momentum and energy are

$$cp = \sqrt{W^2 + 2mc^2W}, \quad E = mc^2 + W. \tag{3}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kp/Q \tag{4}$$

where $k = 10^9/299792458$ and p is the momentum in units of GeV/c. The relativistic parameters β and γ, and the revolution frequency of the ion are

$$\beta = cp/E, \quad \gamma = E/(mc^2), \quad f = c\beta/(2\pi R). \tag{5}$$

Here R is the machine radius, defined to be the closed orbit circumference divided by 2π. The angular frequency is $\omega = 2\pi f$. We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{6}$$

where γ_t is the transition gamma.

1.2 RF Parameters

Various RF Bucket and Bunch parameters are as follows. The half-height of the bucket is

$$\Delta E = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s}{2}\right|^{1/2} \tag{7}$$

where the subscripts "s" indicate parameter values for the synchronous particle, h is the RF harmonic number, ϕ_s is the synchronous phase, and

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2}. \tag{8}$$

Here V_g is the total RF gap voltage per turn and A_S is the area of the corresponding Stationary bucket. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{9}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

The width of the bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{10}$$

where the phase ϕ_e satisfies

$$\cos\phi_e - \cos(\pi - \phi_s) = -\{\phi_e - (\pi - \phi_s)\}\sin\phi_s. \tag{11}$$

The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s}\left\{\frac{-h\eta_s eQV_g\cos\phi_s}{2\pi E_s}\right\}^{1/2} \tag{12}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$.

The half-height and full width of the bunch matched to the bucket are given by

$$\Delta E_m = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{\cos\phi_m - \cos\phi_s + (\phi_m - \phi_s)\sin\phi_s}{2}\right|^{1/2} \tag{13}$$

and

$$\Delta t = \frac{|\phi_m - \phi_e|}{h\omega_s} \tag{14}$$

where the phase ϕ_e satisfies

$$\cos\phi_m - \cos\phi_e + (\phi_m - \phi_e)\sin\phi_s = 0. \tag{15}$$

For a bunch matched to a stationary bucket the half-height and width are given by

$$\Delta E_m = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{\cos\phi_m \mp 1}{2}\right|^{1/2}, \quad \Delta t = \frac{|2\phi_m|}{h\omega_s} \tag{16}$$

where the "$-$" and "$+$" signs are for buckets below and above transition respectively. The area of a small bunch in a stationary bucket is approximately

$$A_b = \left(\frac{\pi A_S}{16}\right)\phi_m^2. \tag{17}$$

2 Lattice Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	C_r	m
C_E	$C_a/4$	$4C_r/19$	C_r	m
ρ	13.8656	85.378351	242.7806	m
γ_t	4.806	8.5	22.89	
Q_H, Q_V	4.757, 4.777	8.78, 8.72	28.19, 29.18	
Max β_H, β_V	13.5, 13.2	22.3, 22.2	48.6, 47.4	m
Max D_H	2.90	2.17	1.81	m

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{18}$$

meters. Note that $4C_r/19 = 2\pi(128.45798)$ which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [5]. The other Booster and AGS parameters were obtained from MAD runs. The RHIC parameters are taken from Ref. [6] and from MAD runs by Steve Tepikian. (The maximum β_H, β_V, D_H listed for RHIC are the maxima in the arcs.)

3 Gold Parameters in Booster, AGS, and RHIC

The parameters values in the following tables are calculated assuming that:

1. The magnetic rigidity of the Au^{32+} ions at Booster injection is $B\rho = 0.8538$ Tm.

2. The magnetic rigidity of the Au^{32+} ions at Booster extraction is $B\rho = 9.1360$ Tm.

3. The magnetic rigidity of the Au^{77+} ions at AGS injection is $B\rho = 3.7216$ Tm.

4. The magnetic rigidity of the Au^{79+} ion at RHIC injection is the same as that of a proton with γ_p such that $G\gamma_p = 46.5$. Here $G + 1 = 2.792847337(29)$ and the proton mass is $m_p = 0.938271998(38)$ GeV/c^2 as reported in Ref. [7]. Thus $\gamma_p = 25.93639684$ and the proton momentum and energy are $P_p = m_p c\sqrt{\gamma_p^2 - 1} = 24.3173002$ GeV/c and $E_p = m_p c^2 \gamma_p = 24.3353949$ GeV. The rigidity is then $B\rho = kP_p = 81.1137824$ Tm.

5. The energy of the Au^{79+} ion at RHIC Store is 100 GeV per nucleon.

The Bunch and Bucket parameters were obtained from the Computer Program "bbat".

In the following tables, more digits are given for some parameters than would be warranted by the measurement precision; this is done for computational convenience. The notation "/N" in the Units column means "per nucleon".

3.1 Gold in Booster

Parameter	Injection	Extraction	Unit
Q	32	32	
m	183.456812	183.456812	GeV/c^2
W	182.7568/197	100.8160	MeV/N
cp	41.57771	444.8981	MeV/N
E	0.9321806	1.0320689	GeV/N
$B\rho$	0.8538	9.1360	Tm
β	0.04460264	0.43107403	
$\gamma - 1$	0.996184/1000	0.108258	
η	-0.955	-0.771	
ϵ_H (95%)	8.3π	8.3π	mm mrad
ϵ_V (95%)	3.9π	3.9π	mm mrad
h	6	6	
hf	0.397607	3.842917	MHz
R	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during multi-turn injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parameter	Injection	Extraction	Unit
No. of Bunches	6	6	
Bunch Spacing	2515.044	260.219	ns
Ions/Bunch	3.0/6	2.4/6	10^9
Bunch Area	0.045/6	0.045/6	eV s/N
Bunch Δt	1500	48	ns
Bunch ΔE	0.65	20	MeV
Bucket ΔE	0.81	51	MeV
Gap Volts	0.5	30	KV
Bucket Area	0.079/6	0.350/6	eV s/N

Capture of the injected beam occurs on a 6 ms porch at constant field. During this time the gap voltage is increased from 0 to 0.5 kV. The bunch area is determined from the measured bunch width at extraction with $\dot{B} = 37$ G/ms and $V_g = 30$ kV.

3.2 Gold in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	77	77	77	
m	183.434144	183.434144	183.434144	GeV/c^2
W	0.0970601	6.983533	8.864878	GeV/N
cp	0.4360888	7.859708	9.751662	GeV/N
E	1.0281979	7.914671	9.796016	GeV/N
$B\rho$	3.7216	67.075078	83.221098	Tm
β	0.42412920	0.99305547	0.99547224	
γ	1.1042382	8.5000	10.520480	
η	-0.806	0.0	0.00481	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	24	12	12	
hf	3.781006	4.426421	4.437007	MHz
R	128.4526	128.4526	128.45798	m

Parameter	Injection	Extraction	Unit
No. of Bunches	24	4	
Bunch Spacing	264.480	676.131	ns
Ions/Bunch	1.4/6	1.3	10^9
Bunch Area	0.180/6	0.270	eV s/N
Bunch Δt	62	15.4	ns
Bunch ΔE	62	2206	MeV
Bucket ΔE	172	20600	MeV
Gap Volts	320	260	KV
Bucket Area	1.76/6	30	eV s/N

During this running period we will initially inject four loads of six bunches into AGS each AGS cycle as we have in the past. Later a new scheme will be commissionned in which the six bunches in Booster are merged into three and then squeezed into adjacent $h = 6$ buckets. This will allow for eight transfers of three bunches to AGS and will double the intensity in

AGS. The 24 bunches from Booster (four loads of six, or eight loads of three) are injected at constant field into stationary $h = 24$ buckets in AGS. The beam is then debunched adiabatically and rebunched at harmonic 4. Acceleration to top energy occurs on harmonic 12. The bunches are extracted on flat-top at constant field.

Upon passing through the carbon stripping foil between the Booster and AGS, the longitudinal emittance increases by about a factor of four due to energy straggling in the foil. An additional increase of 50% results from filamentation in the $h = 24$ buckets on the AGS injection porch. This gives a total increase of a factor of six which is reflected in the tablulated longitudinal parameters.

For this running period, new more uniform stripping foils have been installed in the BTA line. These are expected to reduce the energy straggling by a factor of two or three.

3.3 Gold in RHIC

Parameter	Injection	Transition	Store	Unit
Q	79	79	79	
m	183.433122	183.433122	183.433122	GeV/c^2
W	8.864829	20.382493	99.068867	GeV/N
cp	9.751608	21.293276	99.995665	GeV/N
E	9.795961	21.313625	100.000000	GeV/N
$B\rho$	81.1137824	177.117274	831.763013	Tm
β	0.99547224	0.99904526	0.99995665	
γ	10.520480	22.8900	107.396090	
η	-0.00713	0.0	0.00182	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	2520	
hf	28.023204	28.123787	197.0461006	MHz
$2\pi R$	3833.845181	3833.845181	3833.845181	m

Parameter	Injection	Store	Unit
No. of Bunches	60	60	
Bunch Spacing	214.108	213.148	ns
Ions/Bunch	1.3	1.3	10^9
Bunch Area	0.270	0.7	eV s/N
Bunch Δt	15.5	3.0	ns
Bunch ΔE	2240	30300	MeV
Bucket ΔE	3570	36000	MeV
Gap Volts	340	6000	KV
Bucket Area	0.82	1.18	eV s/N

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